Law of Universal Gravitation 万有引力定律:

Newton's law of Gravitation 万有引力定律:

$$\vec{F}_{12}=-\frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}\qquad G=6.67\times10^{-11}N\cdot m^2/kg^2$$

在地球表面 $F_g=\frac{GM_Em}{R_e^2}=ma_g\Rightarrow a_g=g=\frac{GM_E}{R_e^2}=9.8m/s^2$

Gravitational Potential Energy

$$E_p(r)=-\frac{GMm}{r}$$

The infinity is set as a reference point of $U=0$.

Gravitational Field

Definition: $\vec{g}=\frac{\vec{F}_g}{m}$

Due to Earth: $\vec{g}(r)=\frac{\vec{F}_g}{m}=-\frac{GM_E}{r^2}\hat{r}$

Due to a thin, spherical shell of radius R $\begin{cases}\vec{g}(r)=-\frac{GM}{r^2}\hat{r} & (r>R)\\ \vec{g}=0 & (r<R)\end{cases}$

Kepler's Laws

Law 1: All of the planets move in elliptical orbits with the Sun at one focus.

Law 2: A line joining any planet to the Sun sweeps out equal areas in equal times.

$$\frac{1}{2}|dl_1|r_1=\frac{1}{2}|dl_2|r_2\quad\xRightarrow{|v|=\frac{|dl|}{dt}}\quad|v_1|r_1=|v_2|r_2$$

Law 3: The square of the period of any planet is proportional to the cube of the planet's mean distance from the Sun:

$$T^2=Cr^3$$

$$T^2=\frac{4\pi^2}{GM}r^3\quad if\ M\gg m$$

开普勒定律记忆方法: 1 椭;2 面; Cr^3

解题思路	一定分清r和r^2
$\begin{cases}\text{圆轨道}\rightarrow & F=m\frac{v^2}{r}\\ \text{椭圆轨道}\rightarrow & \begin{cases}Law\ 2\\ law\ 3\ T^2=Cr^3\end{cases}\end{cases}$	$\begin{cases}\text{万有引力}\rightarrow & F=F_g=-\frac{Gm_1m_2}{r^2}\\ \text{引力势能}\rightarrow & E_p(r)=-\frac{GMm}{r}\\ \text{向心加速度}\rightarrow & F=m\frac{v^2}{r}\end{cases}$

Exercise 15: In the "*Return of the Jedi*" Luke Skywalker asked an Ewok to fire a large drift bottle from a space station. The mass of the space station is 400,000 ton and the equivalent radius of it is 20m. The mass of the drift bottle is 10kg. ($G=6.67\times10^{-11}N\cdot m^2/kg^2$)To make sure that the drift bottle can reach as far as possible to escape the gravitation of the space station (fly to infinity),

(a) what is the work must be done on it?

Known: Mass of the space station: $M_s=400{,}000\ ton=4\times10^8kg$

Mass of the bottle: $m_b=10kg$

Radius of the space station: $r_0=20\ m$

Law of Universal Gravitation: $F(r)=\frac{GM_sm_b}{r^2}$

$$W(r)=\int_{r0}^{r}F_rdr=\int_{r0}^{r}\frac{GM_sm_b}{r^2}dr=\left(GM_sm_b\frac{-1}{r}\right)_{r0}^{r}=GM_sm_b\left(\frac{1}{r_0}-\frac{1}{r}\right)$$

$$\lim_{r\to\infty}W(r)=\lim_{r\to\infty}\left[GM_sm_b\left(\frac{1}{r_0}-\frac{1}{r}\right)\right]=\frac{GM_sm_b}{r_0}=\frac{(6.67\times10^{-11})(4\times10^8)(10)}{20}$$

$$=0.01334\ (J)$$

(b) Find the minimum initial velocity (v_0) of it.

Work= kinetic energy

$$\Rightarrow W=\frac{1}{2}m_bv_0^2\Rightarrow v_0=\sqrt{\frac{2W}{m_b}}=\sqrt{\frac{2\times0.01334}{10}}=0.052\left(\frac{m}{s}\right)$$

Haha, Not a hard work.

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Newton's law of Gravitation 万有引力定律:

$\vec{F}_{12}$ $\quad G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$

在地球表面 $F_g =\quad = \quad \Rightarrow a_g = g = \quad = 9.8m/s^2$

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$E_p(r) =$

The infinity is set as a reference point of $U = 0$.

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解题思路	一定分清r和r^2
圆轨道 → 椭圆轨道 →	万有引力 → $F = F_g =$ 引力势能 → $E_p(r) =$ 向心加速度 → $F =$

Exercise 15: In the "*Return of the Jedi*" Luke Skywalker asked an Ewok to fire a large drift bottle from a space station. The mass of the space station is 400,000 ton and the equivalent radius of it is 20m. The mass of the drift bottle is 10kg. ($G = 6.67 \times 10^{-11} N \cdot m^2/kg^2$)To make sure that the drift bottle can reach as far as possible to escape the gravitation of the space station (fly to infinity),

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Known: Mass of the space station: $M_s = 400{,}000\ ton = 4 \times 10^8 kg$

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Radius of the space station: $r_0 = 20\ m$

Law of Universal Gravitation: $F(r) = \frac{GM_s m_b}{r^2}$

$$W(r) = \int_{r0}^{r} F_r dr = \int_{r0}^{r} \frac{GM_s m_b}{r^2} dr = \left(GM_s m_b \frac{-1}{r}\right)_{r0}^{r} = GM_s m_b \left(\frac{1}{r_0} - \frac{1}{r}\right)$$

$$\lim_{r\to\infty} W(r) = \lim_{r\to\infty} \left[GM_s m_b \left(\frac{1}{r_0} - \frac{1}{r}\right)\right] = \frac{GM_s m_b}{r_0} = \frac{(6.67 \times 10^{-11})(4 \times 10^8)(10)}{20}$$

$$= 0.01334\ (J)$$

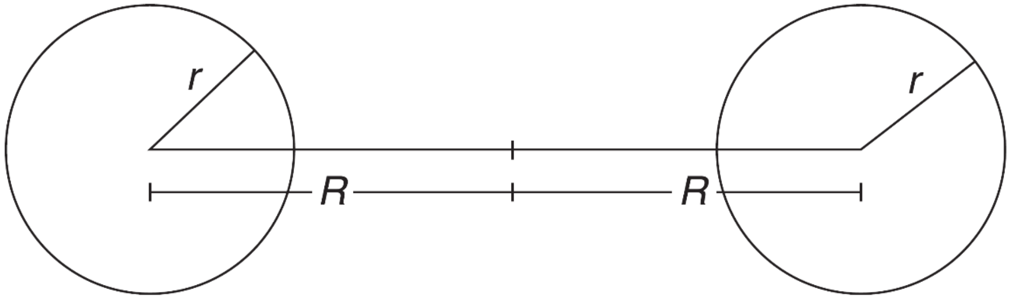
(d) Find the minimum initial velocity (v_0) of it.

Work= kinetic energy

$$\Rightarrow W = \frac{1}{2} m_b v_0^2 \Rightarrow v_0 = \sqrt{\frac{2W}{m_b}} = \sqrt{\frac{2 \times 0.01334}{10}} = 0.052 \left(\frac{m}{s}\right)$$

Haha, Not a hard work.

题型 1 分清r和r^2 分清 R 和 r, 分清 r 和 2r

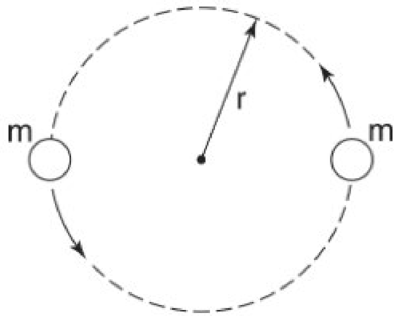


Two stars, each of mass M, form a binary system. The stars orbit about a point a distance R from the center of each star, as shown in the diagram above. The stars themselves each have radius r.

(a) What is the force each star exerts on the other?

(b) In terms of each star's tangential speed v, what is the centripetal acceleration of each star?

Consider the two-star system shown in Figure 10.7, which consists of two stars of mass m rotating in a circle of radius r about their center of mass. What is the total energy of the two-star system? (Choose the gravitational potential energy to be zero when the stars are infinitely far apart from each other, as usual.)



(A) $-Gm^2/2r$

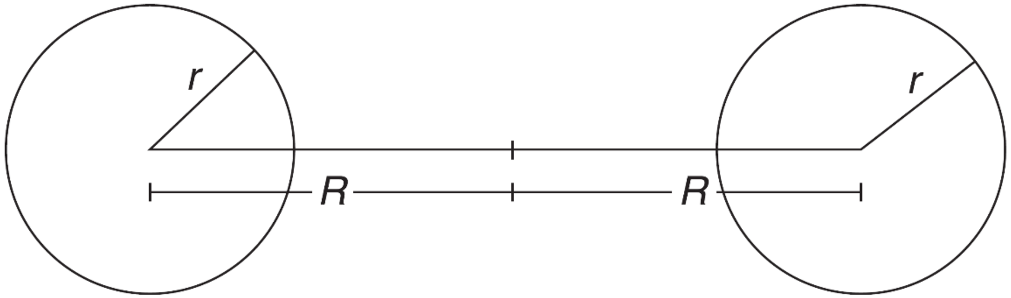
(B) $Gm^2/2r$

(C) $Gm^2/4r$

(D) $3Gm^2/4r$

(E) $-Gm^2/4r$

题型 1 分清r和r^2 分清 R 和 r, 分清 r 和 2r



Two stars, each of mass M, form a binary system. The stars orbit about a point a distance R from the center of each star, as shown in the diagram above. The stars themselves each have radius r.

(a) What is the force each star exerts on the other?

$$G\frac{M^2}{4R^2}$$

(b) In terms of each star's tangential speed v, what is the centripetal acceleration of each star?

$$\frac{v^2}{R}$$

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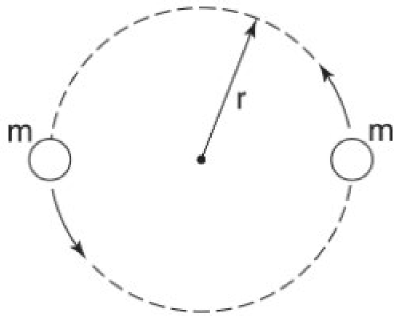


(A) $-Gm^2/2r$
(B) $Gm^2/2r$
(C) $Gm^2/4r$
(D) $3Gm^2/4r$
(E) $-Gm^2/4r$

$E_p=-\frac{GMm}{r}$ 此时r应该取两个star之间的距离2r,且M=m

$$E_p=-\frac{Gm^2}{2r}$$

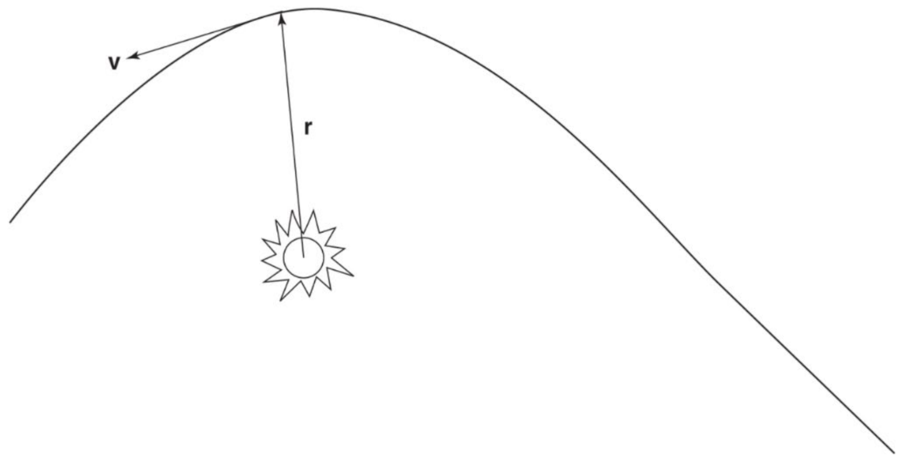
$$m\frac{v^2}{r}=\frac{GMm}{(2r)^2}\Rightarrow v^2=\frac{GM}{4r}$$

$$E_k=\frac{1}{2}mv^2=\frac{1}{2}\frac{Gm^2}{4r}=\frac{1}{2}\frac{Gm^2}{4r}$$

Two stars so that the total $E_k=\frac{Gm^2}{4r}$ 这一步经常遗忘

$E=E_k+E_p=\frac{Gm^2}{4r}-\frac{Gm^2}{2r}=-\frac{Gm^2}{4r}$ (E)

题型 2 逃逸速度相关



An asteroid moves around the sun in a path as shown in the figure above. The asteroid is an unbound particle (and is free to continue an infinite distance away from the sun). At the point indicated, which of the following statements is true of the asteroid's velocity?

(A) $v < \sqrt{\frac{2Gm_{ast}m_{sun}}{r^2}}$ (B) $v > \sqrt{\frac{2Gm_{ast}m_{sun}}{r^2}}$ (C) $v < \sqrt{\frac{Gm_{sun}}{r}}$

(D) $v = \sqrt{\frac{Gm_{sun}}{r}}$ (E) $v > \sqrt{\frac{2Gm_{sun}}{r}}$

A planet revolves in a circular orbit around the sun. Which of the following relationships is true regarding the planet's kinetic and potential energies?

(A) $E_p = -2E_k$ (B) $E_p = -E_k$ (C) $E_p = E_k/2$

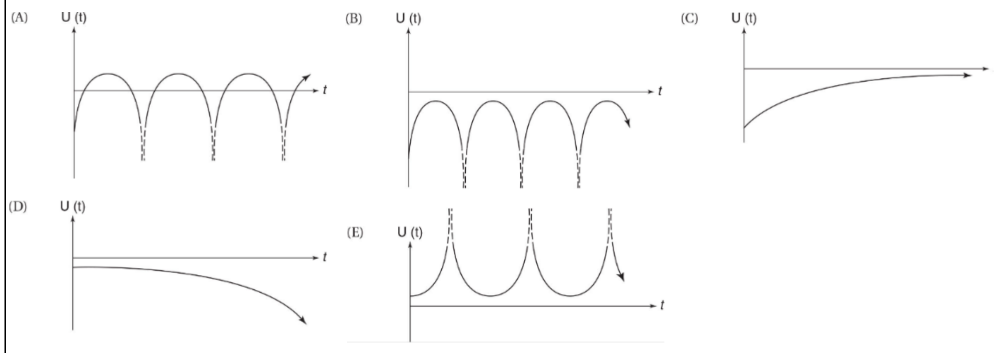
(D) $E_p = -E_k/2$ (E) not enough information is given.

On a planet of radius r and mass m, what is the escape velocity (i.e., the minimum velocity a particle must have to escape the gravitational field of the planet)?

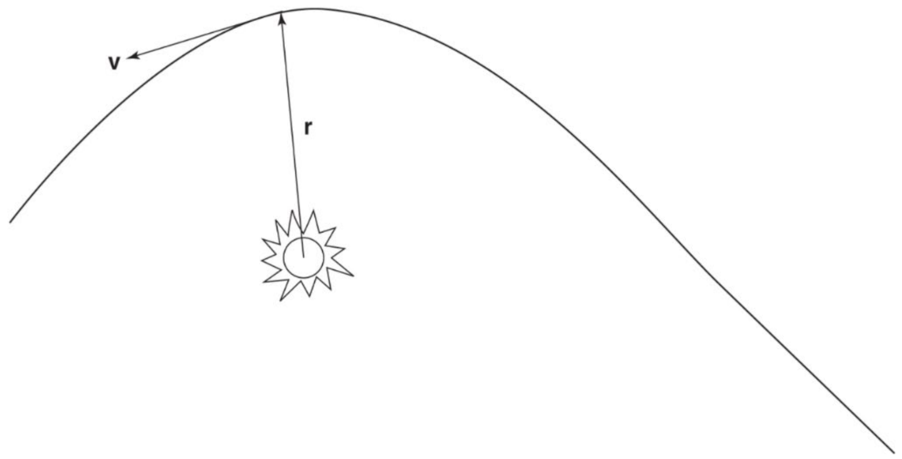
(A) $v = \sqrt{\frac{Gm}{r}}$ (B) $v = \sqrt{\frac{2Gm}{r}}$ (C) $v = \sqrt{\frac{Gm}{r^2}}$ (D) $v = \sqrt{\frac{Gm}{2r^2}}$

(E) The velocity depends on the mass of the escaping particle.

Consider a two-mass system where one mass is fixed and the second mass is initially moving directly away from the fixed mass. If this constitutes a bound system, such that the second mass does not have sufficient velocity to escape to an infinite distance from the fixed mass, which of the graphs in Figure 10.9 represents the gravitational potential energy of the system as a function of time? (Treat the two masses as points that essentially have no volume and so can "pass through" each other.)



题型 2 逃逸速度相关



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(A) $v < \sqrt{\frac{2Gm_{ast}m_{sun}}{r^2}}$ (B) $v > \sqrt{\frac{2Gm_{ast}m_{sun}}{r^2}}$ (C) $v < \sqrt{\frac{Gm_{sun}}{r}}$

(D) $v = \sqrt{\frac{Gm_{sun}}{r}}$ (E) $v > \sqrt{\frac{2Gm_{sun}}{r}}$

$\because$ free to continue an infinite distance away from the sun:

$\rightarrow$ Total energy greater than 0

$\rightarrow E_k - |E_p| > 0 \Rightarrow \frac{1}{2}m_{ast}v^2 - \left(\frac{2Gm_{ast}m_{sun}}{R}\right) > 0 \Rightarrow v > \sqrt{\frac{2Gm_{sun}}{r}}$ (E)

A planet revolves in a circular orbit around the sun. Which of the following relationships is true regarding the planet's kinetic and potential energies?

(A) $E_p = -2E_k$ (B) $E_p = -E_k$ (C) $E_p = E_k/2$

(D) $E_p = -E_k/2$ (E) not enough information is given.

$$E_p = -\frac{GMm}{r}$$

$$E_k = \frac{1}{2}mv^2 \qquad (1)$$

$$m\frac{v^2}{r} = \frac{GMm}{r^2} \Rightarrow v^2 = \frac{GM}{r} \quad \text{plug into (1)}$$

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM}{r} = \frac{1}{2}\frac{GMm}{r} = -\frac{1}{2}E_p \qquad \text{(A)}$$

On a planet of radius r and mass m, what is the escape velocity (i.e., the minimum velocity a particle must have to escape the gravitational field of the planet)?

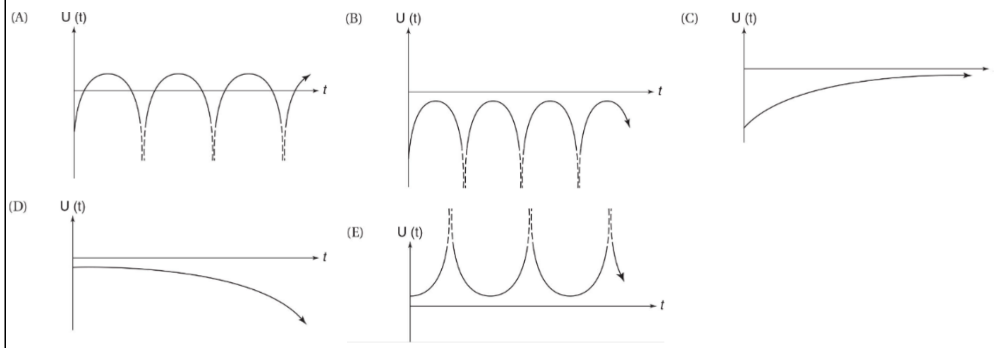
(A) $v = \sqrt{\frac{Gm}{r}}$ (B) $v = \sqrt{\frac{2Gm}{r}}$ (C) $v = \sqrt{\frac{Gm}{r^2}}$ (D) $v = \sqrt{\frac{Gm}{2r^2}}$

(E) The velocity depends on the mass of the escaping particle.

Escape velocity $\rightarrow E_k + E_p = 0$

$$\left.\begin{array}{l} E_p = -\dfrac{GMm}{r} \\ E_k = \dfrac{1}{2}mv^2 \end{array}\right\} \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = 0 \Rightarrow v = \sqrt{\frac{2Gm}{r}} \qquad \text{(B)}$$

Consider a two-mass system where one mass is fixed and the second mass is initially moving directly away from the fixed mass. If this constitutes a bound system, such that the second mass does not have sufficient velocity to escape to an infinite distance from the fixed mass, which of the graphs in Figure 10.9 represents the gravitational potential energy of the system as a function of time? (Treat the two masses as points that essentially have no volume and so can "pass through" each other.)



(B)

题型 3 开普勒定律

A planet moves in an elliptical orbit around the sun. Which of the following statements is true?

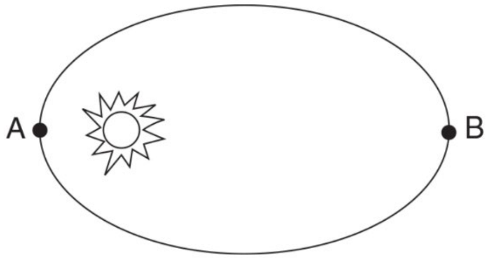
I.The angular momentum of the planet around the sun is constant.

II.The speed of the planet is constant.

III.The total energy (potential plus kinetic) of the planet is constant.

(A)I only (B)II only (C)I and II only (D)I and III only (E)I, II, and III

Consider a planet's orbit around the sun, as shown in the right figure. Which of the following comparisons are valid concerning the planet's speed and angular momentum?



(A) $L_A = L_B$; $v_A > v_B$

(B) $L_A = L_B$; $v_A < v_B$

(C) $L_A > L_B$; $v_A > v_B$

(D) $L_A < L_B$; $v_A < v_B$

(E) $L_A > L_B$; $v_A = v_B$

A mass is moved by a nongravitational force (of unknown magnitude and direction) from point A to point B within a known gravitational field. Which of the following is true?

(A)The change in the object's kinetic energy depends only on its mass and the of points A and B.

(B)The change in the object's kinetic energy depends only on its mass and the path taken between points A and B.

(C)The change in the object's kinetic energy is equal to the work done by the external force, W = ∫F•dr.

(D)The total energy (kinetic plus potential) of the mass is constant.

(E)The object's potential energy depends only on its mass and the locations of points A and B.

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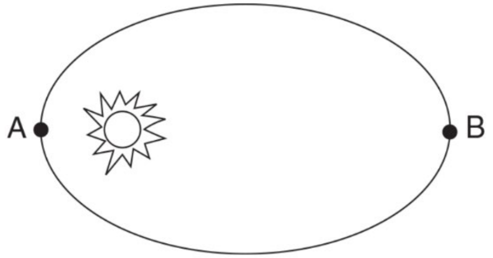
Because the gravitational force is always parallel to the position vector from the sun to the planet (as shown in the derivation of Kepler's second law), the torque is zero and the angular momentum of the planet is constant. I is correct. No external force and internal force is coservertive force, so that the total energy is constant. III is correct.

(D)

Consider a planet's orbit around the sun, as shown in the right figure. Which of the following comparisons are valid concerning the planet's speed and angular momentum?



(A) $L_A = L_B$; $v_A > v_B$

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(C) $L_A > L_B$; $v_A > v_B$

(D) $L_A < L_B$; $v_A < v_B$

(E) $L_A > L_B$; $v_A = v_B$

- The gravitational force is always parallel to the radial position vector, so the gravitational force exerts no torque and the planet orbits with constant angular momentum $L_A = L_B$
- Kepler's 2nd Laws $|v_A|r_A = |v_B|r_B \Rightarrow v_A > v_B$

(A)

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(D)The total energy (kinetic plus potential) of the mass is constant.

(E)The object's potential energy depends only on its mass and the locations of points A and B.

nongravitational force → (A) is wrong

nongravitational force → (B) is wrong

work down changes E_k and E_p → (C) is wrong

nongravitational force → (D) is wrong

E_p depends only on m and location → (E) is wrong

(E)